Determination of the Higgs Boson Mass by the Cancellation of Ultraviolet Divergences in the $SU(2)_L \otimes U(1)$ Theory

Gabriel López Castro
Departamento de Física, CINVESTAV del IPN
Apartado Postal 14-740, México 07000, D.F., México

Jean Pestieau
Institut de Physique Théorique, Université catholique de Louvain
Chemin du Cyclotron 2, B-1348 Louvain-la-Neuve, Belgium

In memory of Roger Decker

Abstract

We assume the vanishing of the quadratic divergences in the $SU(2)_L \otimes U(1)$ electroweak theory. Using the top mass value reported recently by the CDF Collaboration $m_t = 176 \pm 8 \pm 10 GeV$, we predict the mass of the Higgs boson to be $m_H = 321 \pm 29 GeV$. If we assume the vanishing of both quadratic and loga-rithmic divergencies of the top self-mass, we predict $m_t = 170.5 \pm 0.3 GeV$ and $m_H = 308.6 \pm 0.7 GeV$.

1. In the $SU(2)_L \otimes U(1)$ theory, quadratic divergences, at the one loop level, are universal (i.e. they are the same for all physical quantities). We assume they vanish [1]-[4], namely

$$m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) = \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{3}{4}m_H^2 \qquad (1)$$

In the following we shall neglect the fermion masses other than m_t and m_b.
From the recent discovery [5] of the top quark, with a mass

$$\begin{aligned} m_t &= 176 \pm 8 \pm 10 GeV \\ &= 176 \pm 13 GeV \end{aligned} \qquad (2)$$

obtained by the CDF collaboration at FNAL, we predict from Eq (1), the Higgs mass

$$m_H = 321 \pm 29 GeV \qquad (3)$$

where we have used [6]

$$m_Z = 91.1887 \pm 0.0044 GeV \qquad (4)$$

$$m_W = 80.23 \pm 0.18 GeV \qquad (5)$$

2. We assume m_t and m_H to be determined [1-4,7-9] by requiring the ultraviolet divergencies to vanish for the (on-mass shell) self-energy of the quark top, namely

$$\Sigma_t^{\rm div} = -m_t \frac{\alpha}{4\pi} \frac{m_Z^2}{m_Z^2 - m_W^2} F_t(\Lambda) = 0. \qquad (6)$$

for any $\Lambda \cdot F_t(\Lambda)$, the function containing the cutoff Λ has the general form [9]

$$\begin{aligned} F_t(\Lambda) \equiv \frac{3}{4}\frac{1}{m_W^2 m_H^2} & \Big[\Lambda^2 \Big\{m_H^2 + m_Z^2 + 2m_W^2 - 4m_t^2 \\ & -4m_b^2\Big\} + \ln\Lambda^2 \Big\{-\frac{1}{2}m_H^4 - m_Z^4 - 2m_W^4 \\ & +4m_t^4 + 4m_b^4 + \frac{1}{2}m_H^2(m_t^2 - m_b^2) \\ & -\frac{4}{9}m_H^2(m_Z^2 - m_W^2)\Big\}\Big] \end{aligned} \qquad (7)$$

Eqs(6) and (7) imply the two following relations:

$$m_H^2 + m_Z^2 + 2m_W^2 - 4m_t^2 - 4m_b^2 = 0 \tag{8}$$

$$\frac{1}{2}m_H^4 + m_Z^4 + 2m_W^4 - 4m_t^4 - 4m_b^4 - \frac{1}{2}m_H^2(m_t^2 - m_b^2)$$
$$+\frac{4}{9}m_H^2(m_Z^2 - m_W^2) = 0 \tag{9}$$

Of course, Eqs(1) and (8) are identical due to the universality of quadratic divergences [1]-[3]. By using Eqs(4), (5), (8) and (9), we predict:

$$m_t = 170.5 \pm 0.3GeV \tag{10}$$
$$m_H = 308.6 \pm 0.7GeV \tag{11}$$

The predicted mass for the top quark is in agreement with the central value reported by the CDF Collaboration [5], Eq.(2). We have used $m_b = 5GeV$. With $m_b = 0$, our predictions are $m_t = 171.1 \pm 0.3GeV$ and $m_H = 309.7 \pm 0.7GeV$.
Eqs(10) and (11) can be compared with the fitted value of m_t derived from the LEP data [6], $m_t = 173 \pm 13GeV$ if $m_H = 300GeV$. Finally, let us mention that the others solutions to Eqs. (8) and (9) given above namely, $m_t = 78GeV$ and $m_H = 58GeV$, are excluded by the present data. It is interesting to observe that $(2m_t - m_H) \approx 3(m_Z - m_W)$ and $m_t \approx m_Z + m_W$.

References

[1] R. Decker and J. Pestieau, Preprint UCL-IPT-79-19, Université de Louvain (1979); DESY Workshop October 22-24, 1979; Mod. Phys. Lett **A4** (1989) 2733; ibid **A5** (1990) 2579 (E); ibid **A7** (1992) 3773.

[2] M. Veltman Acta Phys. Pol. **B12** (1981) 437.

[3] J. Pestieau, in IIIth Mexican School of Particles and Fields, ed. J.L. Lucio M. and A. Zepeda (World Scientific, 1989), p. 281; in The Gardener of Eden, ed. P. Nicoletopoulos and J. Orloff, Phys. Mag. **12** (1990) 193.

[4] P. Osland and T.T. Wu, Z. Phys. **C55** (1992) 569, 585, 593; Phys. Lett. **B291** (1992) 315; C. Newton, P. Osland and T.T. Wu, Z. Phys. **C61** (1994) 421, 441.

[5] F. Abe *et al.*, CDF Collaboration, *Observation of Top Quark Production in $p\bar{p}$ Collisions*, FERMILAB-PUB-95/022-E (1995).
See also S. Abachi *et al*, D⊘ Collaboration, *Observation of the Top Quark*, FERMILAB-PUB 95/028-E (1995), where $m_t = 199 \pm 30$ GeV is given.

[6] D. Schaile, *Precision Tests of the Electroweak Interaction*, in Proceedings of the XXVII International Conference on High Energy Physics, 20-27 July 1974, Glasgow, Institute of Physics Publishing, Bristol and Philadelphia (1995), p. 27.

[7] J. Pestieau and P. Roy, Phys. Rev. Lett. **23** (1969) 349; R. Decker and J. Pestieau, Lett. Nuovo Cimento **29** (1980) 560.

[8] E.C.G. Stückelberg, Nature **144** (1939) 118; A. Pais, Verh. Roy. Acad. Amsterdam **19** (1946) 1; S. Sakata and O. Hara, Progr. Theor. Phys. **2** (1947) 30; H. Terazawa, Phys. Rev. Lett. **22** (1969) 254, 442 (E); Phys. Rev. **D1** (1970) 2950; ibid **D4** (1971) 1579.

[9] I-Hsiu Lee and S.D. Drell, SLAC-PUB-5423, RU 91-5-B (1991); Bég Memorial Volume.